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History and key aspect of Modern Algebra

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Abstract: Prior to the nineteenth century, algebra meant the study of the solution of polynomial equations. By the twentieth century algebra came to encompass the study of abstract, axiomatic systems such as groups, rings, and fields. This presentation provides an account of the history of the basic concepts, results, and theories of abstract algebra. The development of abstract algebra was propelled by the need for new tools to address certain classical problems that appeared unsolvable by classical means. A major theme of the approach in this book is to show how abstract algebra has arisen in attempts to solve some of these classical problems, providing context from which the reader may gain a deeper appreciation of the mathematics involved. Key features: Begins with an overview of classical algebra Contains separate chapters on aspects of the development of groups, rings, and fields Examines the evolution of linear algebra as it relates to other elements of abstract algebra Highlights the lives and works of six notables: Cayley, Dedekind, Galois, Gauss, Hamilton, and especially the pioneering work of Emmy Noether Offers suggestions to instructors on ways of integrating the history of abstract algebra into their teaching Each chapter concludes with extensive references to the relevant literature Mathematics instructors, algebraists, and historians of science will find the work a valuable reference. The book may also serve as a supplemental text for courses in abstract algebra or the history of mathematics.

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Introduction: Modern algebra, also called **abstract algebra**, branch of mathematic sconcerned with the general algebraic structure of various sets (such as real numbers, complex numbers, matrices, and vector spaces), rather than rules and procedures for manipulating their individual elements. During the second half of the 19th century, various important mathematical advances led to the study of sets in which any two elements can be added or multiplied together to give a third element of the same set. The elements of the sets concerned could be numbers, functions, or some other objects. As the techniques involved were similar, it seemed reasonable to consider the sets, rather than their elements, to be the objects of primary concern. A definitive treatise, *Modern Algebra*, was written in 1930 by the Dutch mathematician Bartel van der Waerden, and the subject has had a deep effect on almost every branch of mathematics.

Basic Algebraic Structures:

Fields: In itself a set is not very useful, being little more than a well-defined collection of mathematical objects. However, when a set has one or more operations (such as addition and multiplication) defined for its elements, it becomes very useful. If the operations satisfy familiar arithmetic rules (such as associativity, commutativity, and distributivity) the set will have a particularly "rich" algebraic structure. Sets with the richest algebraic structure are known as fields. Familiar examples of fields are the rational numbers (fractions a/b where a and b are positive or negative whole numbers), the real numbers (rational and irrational numbers), and the complex numbers (numbers of the form $a + bi$ where a and b are real numbers and $i^2 = -1$). Each of these is important enough to warrant its own special symbol: **Q** for the rationals, **R** for the reals, and **C** for the complex numbers. The term *field* in its algebraic sense is quite different from its use in other contexts, such as vector fields in mathematics or magnetic fields in physics. Other languages avoid this conflict in terminology; for example, a field in the algebraic sense is called a *corps* in French and a *Körper* in German, both words meaning "body."

Structural axioms: The basic rules, or axioms, for addition and multiplication are shown in the table, and a set that satisfies all 10 of these rules is called a field. A set satisfying only axioms 1–7 is called a ring,