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EARLY ALGEBRA AND MATHEMATICS SPECIALISTS

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Early Algebra: What Is It?

The *Principles and Standards for School Mathematics* describes six content standards for grades K-12 [1]. The Algebra Standard envisions students who:

- Understand patterns, relations, and functions;
- Represent and analyze mathematical situations and structures using algebraic symbols;
- Use mathematical models to represent and understand quantitative relationships; and,
- Analyze change in various contexts.

It is important to realize that this Standard spans the elementary and secondary grades. Algebra is a body of knowledge that students learn over a long span of time, beginning in the early grades. Indeed, algebra is not separate from the arithmetic studied in the elementary grades; rather, algebra and arithmetic are integrally connected.

It is also important to understand that early algebra is not what we understand as high school algebra taught in earlier grades. Most researchers echo Carpenter and Levi who claim the goal of early algebra is to develop algebraic thinking [2]. They, like other researchers in the field, conceive of algebraic reasoning as the building, expression, and justification of generalizations, representing mathematical ideas with symbols, and using those symbols to represent and solve problems [3-8]. The algebraic reasoning most appropriate for elementary school that is the focus of these researchers‘ work typically falls into one of two subcategories: *generalized arithmetic* and *functions.*

Generalized Arithmetic—This term refers to the reasoning that occurs as students recognize patterns that emerge during their study of the four basic operations, and to the claims they make and later justify, and eventually express with symbolic notation. For example, a student solving the problem 37 + 28 may take 3 from the 28 and add it to 37; the resulting problem becomes

40 + 25. At first, the student may state a generalization of what he notices as with words: ―When you take an amount from one addend and add the same amount to the other addend, you still get the same total when you add them together.‖ This serves as the basis for the symbolic expression of the relationship, $(a+b) = (a+c) + (b-c)$.

contextual situations, and may be represented with pictures, number lines, function tables symbolic notation, and graphs. For example, six pennies are added to a jar every day and the children analyze the growth.

An essential ingredient of early algebraic instruction is the focus on student reasoning and the discourse that allows students to identify connections among concepts, and then build on these connections to form generalizations. This discourse does not occur naturally, but rather is the result of a well articulated plan, developed by a teacher who herself understands the underlying algebraic aspects of the content. So early algebra is not just appropriate content, but also requires effective pedagogy to bring the deep meaning of the content to the surface.

Why Emphasize Algebra in Elementary Grades?

There are several reasons why an emphasis on early algebra in elementary grades is warranted. First, there is a call for early algebra on both national and state levels. Nationally, there is an emphasis on having all students complete at least one algebra course before graduating from high school. The NCTM released a position paper claiming all students should have an opportunity to learn algebra; furthermore, students need opportunities to encounter algebraic ideas across the PreK-12 curriculum [9]. Statewide, Virginia students are required by the Virginia Department of Education to pass at least three mathematics courses at or above the level of Algebra I in order to obtain a Standard Diploma [10]. The Virginia Department of Education‘s ―Mathematics Standards of Learning‖ require students to explore algebraic concepts in grades K-6 [11]. Some examples of algebra content in these grades include: the formal exploration before sixth grade of the commutative, associative, and distributive properties; an understanding of equality and inequality by second grade; and, the ability to recognize and ―describe a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms‖ by third grade [11].

Another reason to emphasize early algebra in the elementary schools focuses on issues of equity. The Equity Principle states, ―All students need access each year to a coherent, challenging mathematics curriculum taught by competent and well-supported mathematics teachers‖ [1]. Schifter, et al. report that a focus on algebraic representations, generalizations, and connections supports students‘ computational fluency [6]. Furthermore, in the same article they provide evidence that working on developing algebraic reasoning supports the range of learners in a

offered more entry points; more capable students find the content associated with early algebra challenging and stimulating.‖ Thus, a curriculum grounded in early algebra offers greater opportunities for differentiation practices that are focused on substantial mathematical thinking.

A third argument for an emphasis on early algebra revolves around improving overall elementary mathematics curriculum. A curriculum focused on early algebra, with a constant eye on helping children build on past experiences to form generalizations that can be justified, will be much more coherent than a curriculum that ―covers the Standards.‖ A curriculum tied together by algebraic concepts makes sense, and in fact might reduce what seems to be an overwhelming amount of material to learn by providing opportunities to teach more concepts simultaneously [12]. A simple case: understanding the commutative property reduces the number of basic facts one must learn by half. A less simple case: understanding how the distributive property is applied when multiplying whole numbers allows a student to apply the same process when multiplying mixed numbers. Another less simple case: approaching fact instruction through a functional lens creates opportunity for meaningful graphing experiences, tied to pattern exploration and tabular representations.

One aspect of the work on early algebra that seems so promising is that it does not require an entire reworking of the current elementary curriculum. Rather, as Carraher, et al. state, ―existing content needs to be subtly transformed to bring out its algebraic character‖ [7]. Kaput refers to this as ―algebrafying‖ the elementary school curriculum [3]. This ―algebrafication‖ requires ―acknowledging the several different aspects of algebra and their roots in younger children‘s mathematical activity.‖

Enter the Mathematics Specialists

Kaput and Blanton claim ―elementary teachers are in the critical path to longitudinal algebra reform, yet they typically have little experience with the rich and connected activities of generalizing and formalizing‖ [13]. One predictable result of this lack of experience may be a lack of depth of understanding achieved by students, even those who are successful with the *Standards of Learning*. For example, consider two students who are asked to decide if 37 + 52 > 38 +51. Student 1, taught by a teacher without a deep understanding of algebraic concepts, will likely resort to simply adding both sides of the equation, obtaining the same answer, and claiming the statement to be false. This is true, but an opportunity has been missed to use what

given an opportunity to solve this problem in ways that provide initial experiences with commutative and associative properties. Student 2, taught by a teacher with a deep understanding of the concepts and generalizations that can come from this problem, would likely solve this problem in a far different manner than Student 1. Student 2 might reason that 37 is one less than 38, but 52 is one more than 51, so the two sides are still even, using number sense and the relations between the numbers to arrive at a correct answer.

If elementary teachers lack the content and pedagogical knowledge necessary for providing the type of instruction focused on early algebraic reasoning, then clearly this is an area for their professional development. Several groups have reported their efforts in working with teachers as they begin to approach instruction of the elementary mathematics curriculum through an algebraic lens [15-17]. The approaches of these groups reflect the ―algebrafication‖ strategy described by Blanton and Kaput [15]. This strategy is focused on classroom teacher change, approached along three avenues: 1) the ―algebrafication‖ of instructional materials; 2) the support of students' algebraic thinking; and, 3) the creation of a classroom culture and teaching practices supportive of algebraic reasoning.

Mathematics Specialists are in a critical position to provide sustained professional development focused on algebraic reasoning. In their daily work with teachers, Mathematics Specialists regularly work with teachers to plan daily lessons and overall curriculum, work that includes modification of existing instructional resources. In schools with Mathematics Specialists, teachers are becoming better adept at listening to and exploring student reasoning, and helping students build on their own reasoning. As a result of efforts on the part of Mathematics Specialists, more and more teachers afford students opportunities to explore and deeply engage in mathematical explorations, and classroom cultures are established that respect individual reasoning. So, the basic structures of ―algebrafication‖ are in place as a result of Mathematics Specialists in schools.

Yet for ―algebrafication‖ to occur, early algebraic reasoning needs to become a focus of the Mathematic Specialists' work. Specialists need to provide opportunities for the teachers in their school to explore algebraic concepts for themselves in order to gain some depth of understanding of early algebra. As a Specialist works with teachers on lessons and curriculum, for example, the focus can be on underlying algebraic aspects of the concept in question, and how those aspects

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